Examples:

1. Graph $y= log\_{2}\left(x+2\right)-3$

* Since we are given the function$ y= log\_{2}\left(x+2\right)-3,$ we first start by identifying the parent function. The parent function is $y= log\_{2}\left(x\right)$. In addition this function has a vertical asymptote, the line $x=0.$ Next graph the asymptote and parent function by constructing a table of values after rewriting the function of “$x$” as a function of “$y$.
* Recall that$ y= log\_{2}\left(x\right) \leftrightarrow 2^{y}=x$, and let $y$ vary in the table of values and solve for$ x,$ lastly plot and connect the points with a smooth curve.

|  |  |
| --- | --- |
| $$x=2^{y}$$ | $$y$$ |
| $$^{1}/\_{4}$$ | -2 |
| $$^{1}/\_{2}$$ | -1 |
| 1 | 0 |
| 2 | 1 |
| 4 | 2 |

* Second look for the transformations inside the function, since we have the function $y= log\_{2}\left(x+2\right)-3,$ inside the function is the expression$ \left(x+2\right)$. Since$ \left(x+2\right)$ is inside the function the fact two is positive tells us to shift the parent function two units to the left. After undergoing a shift to the left all of the $y$ values stay the same. However, all the $x$ values decrease by two; to show this on the table of values, subtract two from each $x$ value in the domain. So

|  |  |
| --- | --- |
| $$x=2^{y}-2$$ | $$y$$ |
| $$-^{7}/\_{4}$$ | -2 |
| $$-^{3}/\_{2}$$ | -1 |
| -1 | 0 |
| 0 | 1 |
| 2 | 2 |

* Finally look for the transformations outside of the function, given $y= log\_{2}\left(x+2\right)-3,$ outside the function is the expression$-3$. Since$-3,$ is outside we need to shift, the image of the parent function which is shifted two units to the left, and three units down. After undergoing a shift down all of the $x$ values stay the same. However, all the $y$ values decrease by three; to show this on the table of values, subtract three to each $y$ value in the range. So,

|  |  |
| --- | --- |
| $$x=2^{\left(y+3\right)}+2$$ | $$y-3$$ |
| $$-^{7}/\_{4}$$ | -5 |
| $$-^{3}/\_{2}$$ | -4 |
| -1 | -3 |
| 0 | -2 |
| 2 | -1 |

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2. Find the equation of the logarithmic function passing through points: $\left(-3,-3\right), \left(-2,-4\right)$and $\left(0,-5\right) $when the asymptote is the line $x=-4.$

* Because we know the asymptote is $x=-4$ this directly implies that our function is $y= log\_{a}\left(x+4\right)+k$. Now we must find $a$ and$ k$ to solve tis problem. To do so we will use the characteristic point of $\left(1, 0\right)$ and see what happens to it under the shift left four units,

$\left(1, 0\right)\rightarrow \left(-3, 0\right)$.

* We see that $\left(-3, 0\right)$ is not in our collection of given points, but $\left(-3, -3\right)$ is. This means we can find the outside transformation by deriving the transformation that takes $\left(-3, 0\right)\rightarrow \left(-3, -3\right)$, it is easy to see that this is a shift down by three units which implies $k=-3, $ and our function becomes$ y= log\_{a}\left(x+4\right)-3$
* Now we need to find $a$, this is accomplished by selecting another point from the givens and rewriting the logarithmic function as an exponential function and solving for $a.$

$$y= log\_{a}\left(x+4\right)-3 \leftrightarrow a^{\left(y+3\right)}=x+4$$

 Using the point $\left(0,-5\right) \rightarrow x=0 and y=-5$ so,

$$a^{\left(-5+3\right)}=0+4 \rightarrow a^{\left(-2\right)}=4 \rightarrow a=\frac{1}{2}$$

$$∴ y= log\_{\frac{1}{2}}\left(x+4\right)-3 $$

* Use the last point in the givens.,$ \left(-2,-4\right)$, and the derived function to check your answer by establishing a tautology.

$$-4\overset{?}{\overbrace{=}} log\_{\frac{1}{2}}\left(-2+4\right)-3 \rightarrow -1\overset{?}{\overbrace{=}} log\_{\frac{1}{2}}\left(2\right) \rightarrow \left(\frac{1}{2}\right)^{-1}\overset{?}{\overbrace{=}} 2 \rightarrow 2\overset{true}{\overbrace{=}}2$$

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