Examples:

1. Graph $y= log\_{3}\left(x-2\right)+3$

* Since we are given the function$ y= log\_{3}\left(x-2\right)+3,$ we first start by identifying the parent function. The parent function is $y= log\_{3}\left(x\right)$. In addition this function has a vertical asymptote, the line $x=0.$ Next graph the asymptote and parent function by constructing a table of values after rewriting the function of “$x$” as a function of “$y$.
* Recall that$ y= log\_{3}\left(x\right) \leftrightarrow 3^{y}=x$, and le $y$ vary in the table of values and solve for$ x,$ lastly plot and connect the points with a smooth curve.

|  |  |
| --- | --- |
| $$x=3^{y}$$ | $$y$$ |
| $$^{1}/\_{9}$$ | -2 |
| $$^{1}/\_{3}$$ | -1 |
| 1 | 0 |
| 3 | 1 |
| 9 | 2 |

* Second look for the transformations inside the function, since we have the function $y= log\_{3}\left(x-2\right)+3,$ inside the function is the expression$ \left(x-2\right)$. Since$ \left(x-2\right)$ is inside the function the fact two is negative tells us to shift the parent function two units to the right. After undergoing a shift to the right all of the $y$ values stay the same. However, all the $x$ values increase by two; to show this on the table of values, add two to each $x$ value in the domain. So,

|  |  |
| --- | --- |
| $$x=3^{y}+2$$ | $$y$$ |
| $$^{19}/\_{9}$$ | -2 |
| $$^{7}/\_{3}$$ | -1 |
| 3 | 0 |
| 5 | 1 |
| 11 | 2 |

* Finally look for the transformations outside of the function, given $y= log\_{3}\left(x-2\right)+3,$ outside the function is the expression$+3$. Since$+3,$ is outside we need to shift, the image of the parent function which is shifted two units to the right, three units up. After undergoing a shift up all of the $x$ values stay the same. However, all the $y$ values increase by three; to show this on the table of values, add three to each $y$ value in the range. So,

|  |  |
| --- | --- |
| $$x=3^{\left(y-3\right)}+2$$ | $$y+3$$ |
| $$^{19}/\_{9}$$ | 1 |
| $$^{7}/\_{3}$$ | 2 |
| 3 | 3 |
| 5 | 4 |
| 11 | 5 |

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2. Find the equation of the logarithmic function passing through points: $\left(-1,-1\right), \left(0,-2\right)$and $\left(2,-3\right) $when the asymptote is the line $x=-2.$

* Because we know the asymptote is $x=-2$ this directly implies that our function is $ y= log\_{a}\left(x+2\right)+k$. Now we must find $a$ and$ k$ to solve tis problem. To do so we will use the characteristic point of $\left(1, 0\right)$ and see what happens to it under the shift left two units, $\left(1, 0\right)\rightarrow \left(-1, 0\right)$.
* We see that $\left(-1, 0\right)$ is not in our collection of given points, but $\left(-1, -1\right)$ is. This means we can find the outside transformation by deriving the transformation that takes $\left(-1, 0\right)\rightarrow \left(-1, -1\right)$, it is easy to see that this is a shift down by one unit which implies $k=-1, $ and our function becomes$ y= log\_{a}\left(x+2\right)-1$
* Now we need to find $a$, this is accomplished by selecting another point from the givens and rewriting the logarithmic function as an exponential function and solving for $a.$

$$y= log\_{a}\left(x+2\right)-1 \leftrightarrow a^{\left(y+1\right)}=x+2$$

 Using the point $\left(0,-2\right) \rightarrow x=0 and y=-2$ so,

$$a^{\left(-2+1\right)}=0+2 \rightarrow a^{\left(-1\right)}=2 \rightarrow a=\frac{1}{2}$$

$$∴ y= log\_{\frac{1}{2}}\left(x+2\right)-1 $$

* Use the last point in the givens and the derived function to check your answer by establishing a tautology.

$$-3\overset{?}{\overbrace{=}} log\_{\frac{1}{2}}\left(2+2\right)-1 \rightarrow -2\overset{?}{\overbrace{=}} log\_{\frac{1}{2}}\left(4\right) \rightarrow \left(\frac{1}{2}\right)^{-2}\overset{?}{\overbrace{=}} 4 \rightarrow \frac{1}{2}\overset{true}{\overbrace{=}}\frac{1}{2}$$

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